2.2: Creating and Solving Equations Notes

Operation	Word
Addition	the sum of, added to, plus, more than, increased by, total, altogether, and
Subtraction	Less than, minus, subtracted from, the difference of, take away, taken from, reduced by
Multiplication	times, multiplied by, the product of, percent of
Division	Divided by, division of, quotient of, divided into, ratio of

I. Creating Equations from Verbal Descriptions

A. Example:

Suppose Cory and his friend Walter go to a movie. Each of their tickets cost the same amount, and they share a frozen yogurt that costs $5.50. The total amount they spend is $19.90. How can you write this as an equation that describes the situation?

1) Identify the Important Information
a) "Is" means the relationship describes an equation
b) "Total" means the relationship involves the operation of addition.
c) Numerical Information?
i) The frozen yogurt costs $5.50
ii) The total amount spent is $19.90.
d) Unknown Quantity?
i) The cost of the ticket.
2) Write a verbal description.
a) Choose a name for the variable.
Use c for cost
b) Verbal Description = Twice the cost of a ticket plus the cost of the frozen yogurt equals the total amount.
3) Write an equation as a numerical or algebraic expression for each quantity and insert an equal sign in the appropriate place.
Equation = $2c + 5.50 = 19.90$

4. Involving the Distributive Property

A. Example: Aaron and Alice are bowling. Alice's score is twice the difference of Aaron's score and 5. The sum of their scores is 320. Find each student's bowling score.

1) Verbal Description

The sum of Aaron's score and Alice's score is 320

2) Choose a variable for the unknown and write an equation to model the detailed situation.

Let a represent Aaron's score. Then $2(a-5)$ represents Alice's score.

Equation $\Rightarrow$ $a + 2(a-5) = 320$

3) Solve the equation for a.

$$a + 2(a-5) = 320$$

$$a + 2a - 10 = 320 \quad \text{Distributive Property}$$

$$3a - 10 = 320$$

$$3a - 10 + 10 = 320 + 10 \quad \text{Addition Property of Equality.}$$

$$3a = 330$$

$$\frac{3a}{3} = \frac{330}{3} \quad \text{Division Property of Equality}$$

$$a = 110.$$

So Aaron's score is 110 and Alice's score $2(a-5) = 2(110-5) = 2(105) = 210$.

B. Example 2: Mari, Carlos, and Amanda collect stamps. Carlos has five more stamps than Mari, and Amanda has three times as many stamps as Carlos. Altogether, they have 100 stamps. Find the number of stamps each person has.

1) Verbal Description

The total of the number of stamps Mari, Carlos, and Amanda have is 100.

2) Choose a variable for the unknown quantity and write an equation to model the detailed situation.

→ Let s represent the number of stamps Mari has. Then Carlos has $s+5$ stamps and Amanda has $3(s+5)$ stamps.

3) Solve the equation for s.

$$s + (s+5) + 3(s+5) = 100$$

$$s + s + 5 + 3s + 15 = 100 \quad \text{Distributive Property}$$

$$5s + 20 = 100 \quad \text{Combine like terms}$$

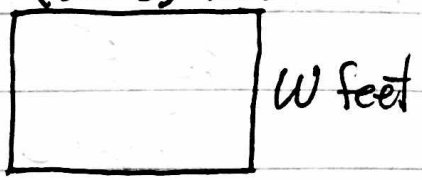
$$5s = 80 \quad \text{Subtraction Property of Equality}$$

$$s = 16 \quad \text{Division Property of Equality}$$

So, Mari has $\boxed{16}$ stamps, Carlos has $\boxed{21}$ stamps, and Amanda has $\boxed{63}$ stamps.

C. Student Example

A rectangular garden is fenced on all sides with 256 feet of fencing. The garden is 8 feet longer than it is wide. Find the length and width of the garden.



$2w + 2(w+8) = 256$

$2w + 2w + 16 = 256$ Distributive Property

~~$2w + 2w$~~ $4w + 16 = 256$ Combining like terms

$4w = 240$ Subtraction Property of Equality

$w = 60$ Division Property of Equality

Width = $\boxed{60 \text{ ft}}$, length = $60 + 8 = \boxed{68 \text{ ft}}$

III. Creating and Solving Equations with Variables on Both Sides

A. Example 1: Janine has job offers at two companies. One company offers a starting salary of $\$28,000$ with a raise of $\$3,000$ each year. The other company offers a starting salary of $\$36,000$ with a $\$2,000$ raise each year.

In how many years would Jamie's salary be the same with both companies? What will the salary be?

1) Write a Verbal Description

Let n represent the number of years it takes for the salaries to be equal.

Base Salary A plus \$3,000 per year raise = Base Salary B plus \$2,000 per year raise

$$28{,}000 + 3{,}000n = 36{,}000 + 2{,}000n$$

$$28{,}000 + 3{,}000n - 2{,}000n = 36{,}000 + 2{,}000n - 2{,}000n \quad \text{Subtraction Property of Equality}$$

$$28{,}000 + 1{,}000n = 36{,}000 \quad \text{Combine like terms}$$

$$28{,}000 + 1{,}000n - 28{,}000 = 36{,}000 - 28{,}000 \quad \text{Subtraction Property of Equality}$$

$$1{,}000n = 8{,}000$$

$$\frac{1{,}000n}{1{,}000} = \frac{8{,}000}{1{,}000} \quad \text{Division Property of Equality}$$

$$n = 8$$

$$28{,}000 + 3{,}000(8) = 36{,}000 + 2{,}000(8)$$

$$52{,}000 = 52{,}000$$

In 8 years, the salaries offered by both will be \$52,000.

B. Example 2: One moving company charges \$800 plus \$16 per hour. Another company charges \$720 plus \$21 per hour. At what number of hours will the charge by both companies be the same? What is the charge?

1) Write a Verbal Description

Let t represent the number of hours that the move takes.

2) Solve the equation

Moving Charge A plus \$16 per hour = Moving Charge B plus \$21 per hour

$$800 + 16t = 720 + 21t$$

$$800 + 16t - 16t = 720 + 21t - 16t$$ Subtraction Property of Equality

$$800 = 720 + 5t$$

$$800 - 720 = 720 - 720 + 5t$$ Subtraction Property of Equality

$$80 = 5t$$

$$\frac{80}{5} = \frac{5t}{5}$$ Division Property of Equality

$$16 = t$$

The charges are the same for the job that takes $\boxed{16}$ hours.

⇒ Substitute the value in the original equation.

$$800 + 16t = 720 + 21t$$

$$800 + 16(16) = 720 + 21(16)$$

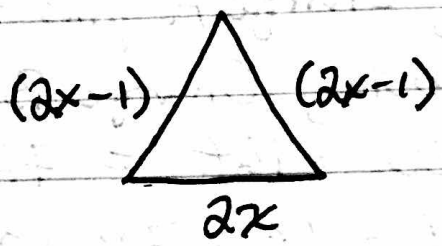
$$800 + 256 = 720 + 336$$

$$1056 = 1056$$

After $\boxed{16}$ hours, the moving charge for both companies will be $\boxed{\$1056}$

C. Student Example 3 & 8

Claire bought just enough fencing to enclose either a rectangular garden of a triangular garden, as shown. The two gardens have the same perimeter. How many feet of fencing did she buy?



$$2(3x-3)+2(x-3) = (2x-1)+(2x-1)+2x$$
$$6x-6+2x-6 = 2x-1+2x-1+2x$$
$$8x-12 = 6x-2$$
$$2x = 10$$
$$x = 5$$

Substitute the value $\boxed{5}$ in the original equation.

$$2(3(5)-3)+2(5-3) = (2(5)-1)+(2(5)-1)+2(5)$$
$$28 = 28$$

Claire bought 28 feet of fencing.

Homework 7

2.1 Pg.	# 1-10, 15-19
2.2 Pg.	# 1-16